

# Form factors of the $B - S$ transitions in the covariant quark model

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In the wake of exploring uncertainty in the full angular distribution of the  $B \rightarrow K\pi + \mu^+\mu^-$  decay caused by the presence of the intermediate scalar  $K_0^*$  meson, we perform the straightforward calculation of the  $B(B_s) \rightarrow S$  ( $S$  is a scalar meson) transition form factors in the full kinematical region within the covariant quark model. We restrict ourselves to the scalar mesons below 1 GeV:  $a_0(980)$ ,  $f_0(500)$ ,  $f_0(980)$ , and  $K_0^*(800)$ . As an application of the obtained results we calculate the widths of the semileptonic and rare decays  $B(B_s) \rightarrow S\ell\bar{\nu}$ ,  $B(B_s) \rightarrow S\ell\bar{\ell}$  and  $B(B_s) \rightarrow S\nu\bar{\nu}$ . We compare our results with those obtained in other approaches.

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## I Introduction

Recently, much attention has been paid to the rare flavor-changing neutral current decay  $B \rightarrow K^*(\rightarrow K\pi)\mu^+\mu^-$ . One of the reasons for this was the first measurement of form-factor-independent angular observables performed by the LHCb Collaboration [1, 2]. It has been claimed that there is a  $3.7\sigma$  deviation from the Standard Model (SM) prediction for one of the angular observables. Much effort has been spent to explain this deviation by invoking the effects of new physics (NP) (for example, see Refs. [3–9] and references therein). The main emphasis of the above-mentioned papers was on the search for the physical observables that have low sensitivity to the form factors.

In addition to the NP effects, the uncertainties related to the presence of the intermediate scalar resonance  $K_0^*$  decaying into  $K\pi$  have been intensively discussed in the literature [10–16]. A detailed analysis of the  $B \rightarrow K_j^*(\rightarrow K\pi)\mu^+\mu^-$  decay in the higher kaon resonance region was done in Ref. [10]. In many papers, the Breit-Wigner form for the  $K\pi$  mass spectra was used. However, this assumption cannot be justified for the broad scalar resonances like the  $K_0^*(800)$  meson. The improvement of the description was done in Ref. [11] by invoking the chiral perturbation theory for the  $K\pi$  interaction. This issue was also generalized to  $B_s \rightarrow K\pi\ell\bar{\nu}$  in Ref. [12].

As is well-known, short-distance physics is under control in the description of the rare  $B$  decays, whereas the effects of long-distance physics described by the hadronic form factors lead to large uncertainties since they involve nonperturbative QCD. The calculation of the  $B \rightarrow K^*$  transition form factors have been performed in many theoretical approaches and models. We must mention some of them: light-cone QCD sum rules [17], QCD sum rules [18], the lattice-constrained dispersion quark model [19], the simple dipole parametrization [20], perturbative QCD at large recoil region [21], the relativistic quark model [22], and the Dyson-Schwinger equations in QCD [23].

The  $B_s$  and  $D_s$  to  $K_0^*(1430)$  transition form factors were calculated in Ref. [27] within an approach based on QCD sum rules. The form factors for the  $B \rightarrow K_0^*(1430)$  transition have been evaluated in the light-front quark model [28]. The form factors of rare  $B \rightarrow K_0^*(1430)\ell^+\ell^-$  decay were calculated in Ref. [29] within three-point QCD sum rules. The  $B \rightarrow S$  transition form factors have been investigated in the light-cone sum rules approach [30]. The transition form factors of  $B(B_s)$ -mesons decay into a scalar meson were studied in Ref. [31] within the perturbative QCD approach. With these form factors, the decay width and branching ratios of the semileptonic  $B \rightarrow S\ell\bar{\nu}$  and rare  $B \rightarrow S\ell^+\ell^-$  decays have been calculated. The rare semileptonic decays  $B_s \rightarrow [f_0(980), K_0^*(1430)]\ell^+\ell^-$  and  $B_s \rightarrow [f_0(980), K_0^*(1430)]\nu\bar{\nu}$  were investigated in Ref. [32] in the framework of the three-point QCD sum rules. The  $B_s \rightarrow f_0(980)$  transition form factors were computed in Ref. [33] by using light-cone QCD sum rules at leading order in the strong coupling constant and an estimate of next-to-leading-order corrections. A QCD light-cone sum rule was also used to evaluate the  $B_s \rightarrow S$  form factors and  $B_s \rightarrow S\ell\bar{\nu}_\ell, \ell\bar{\ell}$  ( $\ell = e, \mu, \tau$ ) branching ratios in Ref. [34]. The twist-3 light-cone distribution amplitudes (LCDAs) of the scalar mesons were investigated in Ref. [35] within the QCD sum rules. As an application of those twist-3 LCDAs, the  $B \rightarrow S$  transition form factors were studied by introducing proper chiral currents into the correlator.

Recently, the  $B - S$  form factors for two scalar nonet mesons below and above 1 GeV were calculated in Ref. [36] by taking into account the perturbative  $\mathcal{O}(\alpha_s)$  corrections to the twist-2 terms using the light-cone QCD sum rules. They were used in Ref. [37] to study the semileptonic  $B \rightarrow S\ell\bar{\nu}_\ell$  and rare  $B \rightarrow S\ell^+\ell^-, S\bar{\nu}\nu$  decays.

In the wake of exploring uncertainty in the full angular distribution of the  $B \rightarrow K^*(\rightarrow K\pi)\mu^+\mu^-$  decay caused by the presence of the intermediate scalar  $K_0^*$  meson, we perform the straightforward calculation of the  $B(B_s) \rightarrow S$  ( $S$  is a scalar meson) transition form factors in the full kinematical region within the covariant quark model. We restrict ourselves to the scalar mesons below 1 GeV:  $a_0(980)$ ,  $f_0(500)$ ,  $f_0(980)$ , and  $K_0^*(800)$  [24]. Actually, the

internal structure of these mesons is not yet well established (see Refs. [25, 26] for a review). We will use the simple  $\bar{q}q$  interpretation of the low-lying scalar mesons in our calculation. The calculated form factors are used to evaluate the branching fractions of the decay  $B(B_s) \rightarrow S\ell\bar{\ell}$ , where  $\ell = e, \mu, \tau$ . We compare our results with those obtained in other approaches.

The paper is organized in the following manner. In Sec. II we give the necessary theoretical framework which includes the effective Hamiltonian, its matrix element between the initial and final states, the definition of the hadronic form factors and the helicity amplitudes. In Sec. III we briefly discuss our covariant quark model and calculate the form factors of the transitions  $B \rightarrow S\ell\nu_\ell$  and  $B \rightarrow S\ell^+\ell^-$ . Finally, we present our numerical results for the differential decay distributions and branching ratios. We compare our findings with the results of other approaches.

## II Effective Hamiltonian and form factors

We start with the on-shell decays  $B_d \rightarrow (K, K_0^*, K^*)\ell^+\ell^-$  which can be described by using the effective Hamiltonian for the  $b \rightarrow s$  transition [38, 39]. The effective Hamiltonian leads to the free-quark  $b \rightarrow s\ell^+\ell^-$  decay amplitude:

$$M(b \rightarrow s\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha\lambda_t}{2\pi} \left\{ C_9^{\text{eff}} (\bar{s}O^\mu b) (\bar{\ell}\gamma_\mu\ell) + C_{10} (\bar{s}O^\mu b) (\bar{\ell}\gamma_\mu\gamma_5\ell) \right. \\ \left. - \frac{2\hat{m}_b}{q^2} C_7^{\text{eff}} (\bar{s}i\sigma^{\mu\nu}(1+\gamma^5)q^\nu b) (\bar{\ell}\gamma_\mu\ell) \right\} \quad (1)$$

where  $O^\mu = \gamma^\mu(1-\gamma^5)$  is the weak Dirac matrix,  $\lambda_t = |V_{ts}^\dagger V_{tb}|$  is the product of the Cabibbo-Kobayashi-Maskawa elements, and  $C_7^{\text{eff}} = C_7 - C_5/3 - C_6$ . The Wilson coefficient  $C_9^{\text{eff}}$  effectively takes into account (i) the contributions from the four-quark operators and (ii) the nonperturbative effects coming from the  $c\bar{c}$ -resonance contributions which are as usual parametrized by a Breit-Wigner ansatz [40]:

$$C_9^{\text{eff}} = C_9 + C_0 \left\{ h(\tilde{m}_c, s) + \frac{3\pi}{\alpha^2} \kappa \sum_{V_i=\psi(1s),\psi(2s)} \frac{\Gamma(V_i \rightarrow l^+l^-) m_{V_i}}{m_{V_i}^2 - q^2 - im_{V_i}\Gamma_{V_i}} \right\} \\ - \frac{1}{2} h(1, s) (4C_3 + 4C_4 + 3C_5 + C_6) \\ - \frac{1}{2} h(0, s) (C_3 + 3C_4) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6) \quad (2)$$

where  $\tilde{m}_c = \hat{m}_c/m_1$ ,  $s = q^2/m_1^2$ ,  $C_0 \equiv 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$  and  $\kappa = 1/C_0$ . Here

$$h(z, s) = -\frac{8}{9} \ln \frac{\hat{m}_b}{\mu} - \frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x \\ - \frac{2}{9} (2+x) |1-x|^{1/2} \begin{cases} \left( \ln \left| \frac{\sqrt{1-x}+1}{\sqrt{1-x}-1} \right| - i\pi \right), & \text{for } x \equiv \frac{4z^2}{s} < 1 \\ 2 \arctan \frac{1}{\sqrt{x-1}}, & \text{for } x \equiv \frac{4z^2}{s} > 1, \end{cases} \\ h(0, s) = \frac{8}{27} - \frac{8}{9} \ln \frac{\hat{m}_b}{\mu} - \frac{4}{9} \ln s + \frac{4}{9} i\pi.$$

where  $\mu$  is a scale parameter and  $m_1 \equiv m_B$ . In what follows, we will not include the long-distance contributions coming from the  $J/\psi$  and  $\psi(2S)$  resonances [40] and charm-loop effects [41].

We specify our choice of the momenta as  $p_1 = p_2 + k_1 + k_2$  with  $p_1^2 = m_1^2$ ,  $p_2^2 = m_2^2$  and  $k_1^2 = k_2^2 = m_\ell^2$  where  $k_1$  and  $k_2$  are the  $\ell^+$  and  $\ell^-$  momenta, and  $m_1, m_2, m_\ell$  are the masses of the initial meson  $H_1$ , the final meson  $H_2$ , and the lepton  $\ell$ , respectively. The matrix elements of the exclusive transitions  $B \rightarrow K(K_0^*)\ell\bar{\ell}$  are defined by

$$M(H_1 \rightarrow H_2\ell\bar{\ell}) = \frac{G_F}{\sqrt{2}} \cdot \frac{\alpha\lambda_t}{2\pi} \cdot \left\{ C_9^{\text{eff}} \langle H_2 | \bar{s}O^\mu b | H_1 \rangle \bar{\ell}\gamma_\mu\ell \right. \\ + C_{10} \langle H_2 | \bar{s}O^\mu b | H_1 \rangle \bar{\ell}\gamma_\mu\gamma_5\ell \\ \left. - \frac{2\hat{m}_b}{q^2} C_7^{\text{eff}} \langle H_2 | \bar{s}i\sigma^{\mu\nu}(1+\gamma^5)q^\nu b | H_1 \rangle \bar{\ell}\gamma_\mu\ell \right\} \quad (3)$$

where  $H_1 = B$ ,  $H_2 = K(K_0^*)$ .

We define dimensionless form factors by

$$\begin{aligned} \langle H_2(p_2) | \bar{s} O^\mu b | H_1(p_1) \rangle &= F_+(q^2) P^\mu + F_-(q^2) q^\mu, \\ \langle H_2(p_2) | \bar{s} i\sigma^{\mu\nu} q_\nu (1 + \gamma^5) b | H_1(p_1) \rangle &= -\frac{1}{m_1 + m_2} (P_\mu q^2 - q_\mu Pq) F_T(q^2), \end{aligned} \quad (4)$$

where  $P = p_1 + p_2$  and  $q = p_1 - p_2$ . The matrix element in Eq (3) is written as

$$M(H_1 \rightarrow H_2 + \bar{\ell}\ell) = \frac{G_F}{\sqrt{2}} \cdot \frac{\alpha\lambda_t}{2\pi} \{T_1^\mu (\bar{\ell}\gamma_\mu \ell) + T_2^\mu (\bar{\ell}\gamma_\mu \gamma_5 \ell)\}$$

where the quantities  $T_i^\mu$  are expressed through the form factors and the Wilson coefficients in the case of the spinless particle  $H_2$  as

$$\begin{aligned} T_i^\mu &= \mathcal{F}_+^{(i)} P^\mu + \mathcal{F}_-^{(i)} q^\mu \quad (i = 1, 2), \\ \mathcal{F}_+^{(1)} &= C_9^{\text{eff}} F_+ + C_7^{\text{eff}} F_T \frac{2\hat{m}_b}{m_1 + m_2}, \\ \mathcal{F}_-^{(1)} &= C_9^{\text{eff}} F_- - C_7^{\text{eff}} F_T \frac{2\hat{m}_b}{m_1 + m_2} \frac{Pq}{q^2}, \\ \mathcal{F}_\pm^{(2)} &= C_{10} F_\pm. \end{aligned} \quad (5)$$

Respectively, the helicity form factors  $H_m^i$  are defined in terms of the invariant form factors as [42]

$$H_t^i = \frac{1}{\sqrt{q^2}} (Pq \mathcal{F}_+^i + q^2 \mathcal{F}_-^i), \quad H_\pm^i = 0, \quad H_0^i = \frac{2m_1 |\mathbf{p}_2|}{\sqrt{q^2}} \mathcal{F}_+^i. \quad (6)$$

The differential  $(q^2, \cos\theta)$  two-fold decay distribution may be written in terms of the bilinear combinations of the helicity amplitudes (see Ref. [42]). However, it is common in the modern literature to use the transversality amplitudes  $A_{\perp, \parallel, 0}^{L,R}$  and  $A_t$  defined in Ref. [43]. They are related to our helicity amplitudes by

$$\begin{aligned} A_\perp^{L,R} &= N \frac{1}{\sqrt{2}} \left[ (H_+^{(1)} - H_-^{(1)}) \mp (H_+^{(2)} - H_-^{(2)}) \right], \\ A_\parallel^{L,R} &= N \frac{1}{\sqrt{2}} \left[ (H_+^{(1)} + H_-^{(1)}) \mp (H_+^{(2)} + H_-^{(2)}) \right], \\ A_0^{L,R} &= N \left( H_0^{(1)} \mp H_0^{(2)} \right), \\ A_t &= -2N H_t^{(2)} \end{aligned} \quad (7)$$

where the overall factor is given by

$$N = \left[ \frac{1}{4} \frac{G_F^2}{(2\pi)^3} \left( \frac{\alpha\lambda_t}{2\pi} \right)^2 \frac{|\mathbf{p}_2| q^2 v}{12m_1^2} \right]^{\frac{1}{2}}$$

where  $|\mathbf{p}_2| = \lambda^{1/2}(m_1^2, m_2^2, q^2)/2m_1$  is the momentum of the outgoing meson  $H_2$  and  $v = \sqrt{1 - 4m_\ell^2/q^2}$  is the lepton velocity, both of which are given in the rest frame of the parent meson  $H_1$ .

The differential decay distribution then reads

$$\begin{aligned}
\frac{d\Gamma(H_1 \rightarrow H_2 \bar{\ell} \ell)}{dq^2 d(\cos \theta)} &= \frac{G_F^2}{(2\pi)^3} \left( \frac{\alpha \lambda_t}{2\pi} \right)^2 \frac{|\mathbf{p}_2| q^2 v}{12m_1^2} \\
&\times \frac{3}{16} \left\{ |H_0^{(1)}|^2 + |H_0^{(2)}|^2 + 2\delta_{\ell\ell} \left[ |H_0^{(1)}|^2 - |H_0^{(2)}|^2 \right] + 4\delta_{\ell\ell} |H_t^{(2)}|^2 \right. \\
&- \cos 2\theta (1 - 2\delta_{\ell\ell}) \left[ |H_0^{(1)}|^2 + |H_0^{(2)}|^2 \right] \left. \right\} \\
&= \frac{3}{8} \left\{ |A_0^L|^2 + |A_0^R|^2 + 4\delta_{\ell\ell} \operatorname{Re} \left( A_0^L A_0^{R\dagger} \right) + 2\delta_{\ell\ell} |A_t|^2 \right. \\
&- \cos 2\theta (1 - 2\delta_{\ell\ell}) \left[ |A_0^L|^2 + |A_0^R|^2 \right] \left. \right\}. \tag{8}
\end{aligned}$$

Integrating over  $\cos \theta$  one obtains

$$\begin{aligned}
\frac{d\Gamma(H_1 \rightarrow H_2 \bar{\ell} \ell)}{dq^2} &= \frac{G_F^2}{(2\pi)^3} \left( \frac{\alpha \lambda_t}{2\pi} \right)^2 \frac{|\mathbf{p}_2| q^2 v}{12m_1^2} \\
&\times \frac{1}{2} \left\{ |H_0^{(1)}|^2 + |H_0^{(2)}|^2 + \delta_{\ell\ell} \left[ |H_0^{(1)}|^2 - 2|H_0^{(2)}|^2 + 3|H_t^{(2)}|^2 \right] \right\} \\
&= (1 - \tfrac{1}{2}\delta_{\ell\ell}) \left[ |A_0^L|^2 + |A_0^R|^2 \right] + \tfrac{3}{2}\delta_{\ell\ell} \left[ 2\operatorname{Re} \left( A_0^L A_0^{R\dagger} \right) + |A_t|^2 \right] \tag{9}
\end{aligned}$$

where we have introduced a flip parameter  $\delta_{\ell\ell} = 2m_\ell^2/q^2$ .

We also calculate the differential rates for the semileptonic  $H_1 \rightarrow H_2 \ell \bar{\nu}_\ell$  mode and rare  $H_1 \rightarrow H_2 \nu \bar{\nu}$  decay. One has

$$\begin{aligned}
\frac{d\Gamma(H_1 \rightarrow H_2 \ell \bar{\nu}_\ell)}{dq^2} &= \frac{G_F^2}{(2\pi)^3} |V_{bu}|^2 \frac{|\mathbf{p}_2| q^2}{12m_1^2} \left( 1 - \frac{m_\ell^2}{q^2} \right)^2 \left\{ \left( 1 + \frac{m_\ell^2}{2q^2} \right) |H_0|^2 + \frac{3m_\ell^2}{2q^2} |H_t|^2 \right\}, \tag{10} \\
H_0 &= \frac{2m_1 |\mathbf{p}_2|}{\sqrt{q^2}} F_+, \quad H_t = \frac{1}{\sqrt{q^2}} (PqF_+ + q^2 F_-),
\end{aligned}$$

$$\frac{d\Gamma(H_1 \rightarrow H_2 \nu \bar{\nu})}{dq^2} = \frac{G_F^2}{(2\pi)^3} \left( \frac{\alpha \lambda_t}{2\pi} \right)^2 \frac{|\mathbf{p}_2|^3}{\sin^4 \theta_W} |D_\nu(x_t)|^2 F_+^2(q^2). \tag{11}$$

The form factors  $F_\pm$  are defined by Eq. (4), whereas the function  $D_\nu(x_t)$  is given by

$$D_\nu(x_t) = \frac{x_t}{8} \left( \frac{2+x_t}{x_t-1} + \frac{3x_t-6}{(x_t-1)^2} \ln x_t \right), \quad x_t = \frac{\hat{m}_t^2}{m_W^2}.$$

### III The $B - S$ transition form factors in the covariant quark model

We calculate the  $B - S$  transition form factors in the covariant quark model. We briefly recall the basic features of this approach, which was formulated in its modern form in Ref. [44] by taking into account the infrared confinement of quarks.

The model is based on an effective interaction Lagrangian describing the coupling of hadrons to their constituent quarks. For instance, the coupling of a meson  $M(q_1 \bar{q}_2)$  to its constituent quarks  $q_1$  and  $\bar{q}_2$  is described by the nonlocal Lagrangian

$$\mathcal{L}_{\text{int}}^{\text{str}}(x) = g_M M(x) \int dx_1 \int dx_2 F_M(x, x_1, x_2) \bar{q}_1(x_1) \Gamma_M q_2(x_2) + \text{H.c.} \tag{12}$$

Here,  $\Gamma_M$  is the Dirac matrix, which is chosen appropriately to describe the spin quantum numbers of the meson field  $M(x)$ . The vertex function  $F_M(x, x_1, x_2)$  characterizes the finite size of the meson. To satisfy translational invariance

the vertex function has to obey the identity  $F_M(x+a, x_1+a, x_2+a) = F_M(x, x_1, x_2)$  for any given four-vector  $a$ . We use a specific form for the vertex function which satisfies the above translation invariance relation. One has

$$F_M(x, x_1, x_2) = \delta^{(4)}(x - \sum_{i=1}^2 w_i x_i) \Phi_M\left((x_1 - x_2)^2\right) \quad (13)$$

where  $\Phi_M$  is the correlation function of the two constituent quarks with masses  $m_1$  and  $m_2$ . The variable  $w_i$  is defined by  $w_i = m_i/(m_1 + m_2)$ , so that  $w_1 + w_2 = 1$ . We choose a simple Gaussian form for the vertex function  $\Phi_M(-k^2)$ . The minus sign in the argument of  $\Phi_M(-k^2)$  is chosen to emphasize that we are working in Minkowski space. One has

$$\tilde{\Phi}_M(-k^2) = \exp(k^2/\Lambda_M^2) \quad (14)$$

where the parameter  $\Lambda_M$  characterizes the size of the meson. Since  $k^2$  turns into  $-k_E^2$  in Euclidean space the form (14) has the appropriate falloff behavior in the Euclidean region. We stress that any choice for  $\Phi_M$  is appropriate as long as it falls off sufficiently fast in the ultraviolet region of Euclidean space in order to render the Feynman diagrams ultraviolet finite.

In the evaluation of the quark-loop diagrams we use the free local fermion propagator for the constituent quark,

$$S_q(k) = \frac{1}{m_q - \not{k} - i\epsilon} \quad (15)$$

with an effective constituent quark mass  $m_q$ .

The coupling constant  $g_M$  in Eq. (12) is determined by the so-called *compositeness condition* suggested by Weinberg [45] and Salam [46] (for a review, see Ref. [47]) and extensively used in our studies (for details, see Ref. [48]). The compositeness condition requires that the renormalization constant  $Z_M$  of the elementary meson field  $M(x)$  is set to zero, i.e.,

$$Z_M = 1 - g_M^2 \Pi'_M(m_M^2) = 0 \quad (16)$$

where  $\Pi'_M(p^2)$  is the derivative of the mass operator corresponding to the self-energy diagram in Fig. 1.

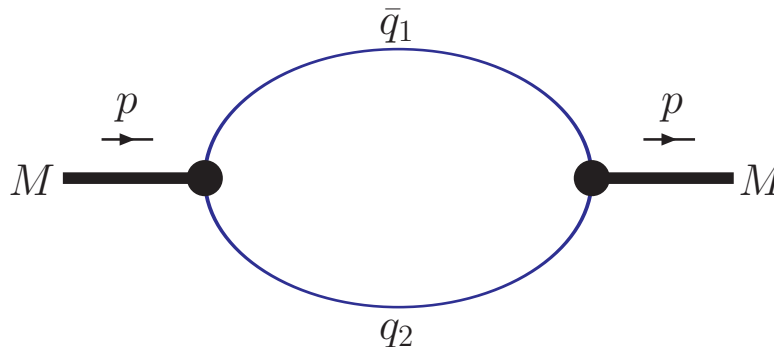


FIG. 1: Diagram describing the meson mass operator.

To clarify the physical meaning of the compositeness condition, we recall that the renormalization constant  $Z_M^{1/2}$  can also be interpreted as the matrix element between the physical state and the corresponding bare state. For  $Z_M = 0$  it then follows that the physical state does not contain the bare one and it is therefore described as a bound state. The interaction Lagrangian (12) and the corresponding free Lagrangian describe both the constituents (quarks) and the physical particles (hadrons), which are bound states of the constituents. As a result of the interaction, the physical particle is dressed, i.e., its mass and wave function have to be renormalized. The condition  $Z_M = 0$  also effectively excludes the constituent degrees of freedom from the space of physical states and thereby guarantees that there will be no double counting. The constituents exist in virtual states only.

The covariant quark model was applied to evaluate the form factors of the  $B(B_s) \rightarrow P(V)$ -transitions in the full kinematical region of momentum transfer squared [49, 50]. This approach was extended to describe the baryons as three-quark states [51] and the exotic meson  $X(3872)$  as a tetraquark [52].

A similar approach based on the compositeness condition  $Z = 0$  was recently developed in Ref. [53].

In this paper we evaluate the  $B-S$  transition form factors assuming that the scalar mesons below 1 GeV are ordinary two-quark states. Some remarks should be made before performing the calculations. The internal structure of the light scalar mesons is not yet well established (for review, see Refs. [25, 26]). Since they have large decay widths it is difficult to distinguish them from background. There are interpretations of these objects as four-quark states and/or gluballs. Here, we describe the scalar mesons as two-quark states and evaluate the  $B-S$  form factors within our approach, but when we use the calculated form factors in the matrix element of the cascade decay  $B \rightarrow K_0^*(\rightarrow K\pi)\ell^+\ell^-$  we take into account the line shape of the  $K_0^*$ , which reflects the broad width of this resonance. One can also describe the scalar mesons as four-quark states in our approach, similar to the exotic meson X(3872) [52]; however, this is beyond the scope of this work.

The SU(3) nonet of scalar mesons below 1 GeV can be written in the matrix form

$$\hat{S} = \frac{1}{\sqrt{2}} \sum_{i=0}^8 S^i \lambda^i \quad \lambda^0 = \sqrt{\frac{2}{3}} I. \quad (17)$$

The physical scalar fields are related to the Cartesian basis in the following manner:

$$\begin{aligned} S^\pm &= \frac{1}{\sqrt{2}}(S^1 \mp iS^2), & S^0 &= S^3, \\ S_s^+ &= \frac{1}{\sqrt{2}}(S^4 - iS^5), & S_s^0 &= \frac{1}{\sqrt{2}}(S^6 - iS^7), \\ S_s^- &= \frac{1}{\sqrt{2}}(S^4 + iS^5), & \bar{S}_s^0 &= \frac{1}{\sqrt{2}}(S^6 + iS^7), \\ S' &= S^0 \cos \theta_S + S^8 \sin \theta_S, & S &= -S^0 \sin \theta_S + S^8 \cos \theta_S, \end{aligned} \quad (18)$$

where  $\theta_S$  is the octet-singlet mixing angle. The  $\bar{q}\hat{S}q$  vertex is then written as

$$\begin{aligned} \mathcal{L}_{S\bar{q}q} &= \bar{q}\hat{S}q \\ &= S^+ \bar{u}d + S^- \bar{d}u + S^0 \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) + S_s^+ \bar{u}s + S_s^0 \bar{d}s + S_s^- \bar{s}u + \bar{S}_s^0 \bar{s}d \\ &+ S' \left( \cos \delta_S \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) - \sin \delta_S \bar{s}s \right) - S \left( \sin \delta_S \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) + \cos \delta_S \bar{s}s \right), \end{aligned} \quad (19)$$

where  $\delta_S = \theta - \theta_I$ , with the ideal mixing angle  $\theta_I = \arctan(1/\sqrt{2})$ . We will use the notation from Ref. [24] for the scalar mesons below 1 GeV:

- $S_s \equiv K_0^*(800)$ ,  $I(J^P) = \frac{1}{2}(0^+)$ ,  $m_{K_0^*(800)} = 682 \pm 29$  MeV;
- $S' \equiv f_0(500)$ ,  $I^G(J^{PC}) = 0^+(0^{++})$ ,  $m_{f_0(500)} = 400 - 550$  MeV;
- $S \equiv f_0(980)$ ,  $I^G(J^{PC}) = 0^+(0^{++})$ ,  $m_{f_0(980)} = 990 \pm 20$  MeV;
- $S^{\pm,0} \equiv a_0^{\pm,0}(980)$ ,  $I^G(J^{PC}) = 1^-(0^{++})$ ,  $m_{a_0(980)} = 980 \pm 20$  MeV.

Moreover, we assume that  $\delta_S = 0$ , i.e.,  $m_{f_0(980)}$  to ensure a pure  $\bar{s}s$  state.

The coupling constant  $g_S$  in Eq. (12) is determined by Eq. (16), where  $\tilde{\Pi}'_S$  is the derivative of the scalar meson mass operator,

$$\begin{aligned} \tilde{\Pi}'_S(p^2) &= -\frac{1}{2p^2} p^\alpha \frac{d}{dp^\alpha} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_S^2(-k^2) \text{tr} \left[ S_1(k + w_1 p) S_2(k - w_2 p) \right] \\ &= -\frac{1}{2p^2} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_S^2(-k^2) \left\{ w_1 \text{tr} \left[ S_1(k + w_1 p) \not{p} S_1(k + w_1 p) S_2(k - w_2 p) \right] \right. \\ &\quad \left. - w_2 \text{tr} \left[ S_1(k + w_1 p) S_2(k - w_2 p) \not{p} S_2(k - w_2 p) \right] \right\}. \end{aligned} \quad (20)$$

By using the calculation technique outlined in Ref. [44], one can easily perform the loop integration. We give the analytic result for equal quark masses ( $m_{q_1} = m_{q_2} \equiv m_q$ ):

$$\begin{aligned} \tilde{\Pi}'_S(p^2) &= \int_0^{1/\lambda^2} \frac{dt}{a_S^2} \int_0^1 d\alpha e^{-t z_0 + z_1} \\ &\times \frac{t}{32} \left\{ p^2 - 4m_q^2 + \frac{1}{a_S} [20 + t(1-2\alpha)^2(12m_q^2 - p^2)] - \frac{t}{a_S^2} (1-2\alpha)^2(12 + p^2 t) + \frac{t^3}{a_S^3} (1-2\alpha)^4 p^2 \right\} \quad (21) \end{aligned}$$

$$z_0 = \alpha m_q^2 - \alpha(1-\alpha)p^2, \quad z_1 = \frac{st}{2a_S} (1-2\alpha)^2 p^2,$$

$$a_S = 2s + t, \quad s = \frac{1}{\Lambda_S^2}.$$

Note that in the case of  $\lambda \rightarrow 0$  the branching point appears at  $p^2 = 4m_q^2$ . At this point the integral over  $t$  becomes divergent as  $t \rightarrow \infty$  because  $z_0 = 0$  at  $\alpha = 1/2$ . By introducing an infrared cutoff on the upper limit of the scale of integration, one can avoid the appearance of the threshold singularity.

Herein our primary subjects are the  $B-S$  transition matrix elements, which can be expressed via the dimensionless form factors defined in Refs. [49, 50]. The diagram corresponding to these matrix elements is shown in Fig. 2.

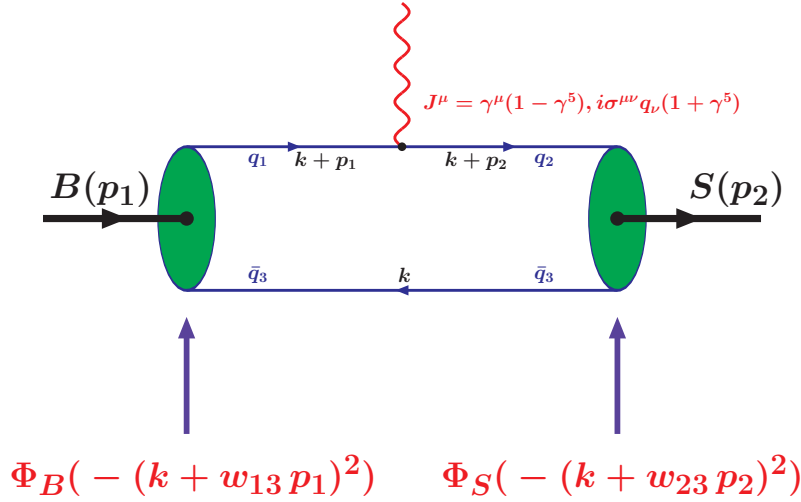


FIG. 2: Diagrammatic representation of the matrix elements in Eqs. (22) and (23).

One has

$$\begin{aligned}
& \langle S_{[\bar{q}_3 q_2]}(p_2) | \bar{q}_2 O^\mu q_1 | B_{[\bar{q}_1 q_3]}(p_1) \rangle \\
&= N_c g_B g_S \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_B \left( -(k + w_{13} p_1)^2 \right) \tilde{\Phi}_S \left( -(k + w_{23} p_2)^2 \right) \\
&\times \text{tr} \left[ S_2(k + p_2) O^\mu S_1(k + p_1) \gamma^5 S_3(k) \right] \\
&= F_+^{BS}(q^2) P^\mu + F_-^{BS}(q^2) q^\mu,
\end{aligned} \tag{22}$$

$$\begin{aligned}
& \langle S_{[\bar{q}_3 q_2]}(p_2) | \bar{q}_2 (i\sigma^{\mu\nu} q_\nu (1 + \gamma^5)) q_1 | B_{[\bar{q}_1 q_3]}(p_1) \rangle \\
&= N_c g_B g_S \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_B \left( -(k + w_{13} p_1)^2 \right) \tilde{\Phi}_S \left( -(k + w_{23} p_2)^2 \right) \\
&\times \text{tr} \left[ S_2(k + p_2) i\sigma^{\mu\nu} q_\nu (1 + \gamma^5) S_1(k + p_1) \gamma^5 S_3(k) \right] \\
&= -\frac{1}{m_1 + m_2} (q^2 P^\mu - q \cdot P q^\mu) F_T^{BS}(q^2).
\end{aligned} \tag{23}$$

Here,  $p_i^2 = m_i^2$ ,  $q_1 = b$ ,  $q_2 = u, s, d$ , and  $q_3 = s, d$ . Since there are three sorts of quarks involved in these processes, we introduce the notation with two subscripts,  $w_{ij} = m_{q_j} / (m_{q_i} + m_{q_j})$  ( $i, j = 1, 2, 3$ ) so that  $w_{ij} + w_{ji} = 1$ .

The first fit of the model parameters was done in the original paper [44], where the infrared quark confinement was implemented for the first time. The leptonic decay constants (which are known either from experiments or from lattice simulations) have been chosen as the input quantities to adjust the model parameters. A given meson  $H$  in the interaction Lagrangian is characterized by the coupling constant  $g_H$ , the size parameter  $\Lambda_H$  and two of the four constituent quark masses,  $m_q$  ( $m_u = m_d$ ,  $m_s$ ,  $m_c$ ,  $m_b$ ). Moreover, there is the infrared confinement parameter  $\lambda$ , which is universal for all hadrons. Note that the physical values for the hadron masses have been used in the fit. Therefore, one has  $2n_H + 5$  adjustable parameters for  $n_H$  numbers of mesons. The compositeness condition provides  $n_H$  constraints and allows one to express all coupling constants  $g_H$  via other model parameters. The remaining  $n_H + 5$  parameters are determined by a fit to experimental data. The values of leptonic decay constants and some electromagnetic decay widths have been chosen as the input data. Several updated fits were done in Refs. [49, 50]. In this paper we will use the latest fit done in Ref. [54]. The fitted values of the constituent quark masses  $m_q$ , the infrared cut-off  $\lambda$ , and the size parameters  $\Lambda_H$  are given by Eq. (24) and Table I.

$m_{u/d}$	$m_s$	$m_c$	$m_b$	$\lambda$	
0.241	0.428	1.67	5.05	0.181	GeV

(24)

TABLE I: The fitted values of the size parameters  $\Lambda_H$  in GeV.

$\pi$	$K$	$D$	$D_s$	$B$	$B_s$	$B_c$	$\eta_c$	$\eta_b$	
0.87	1.02	1.71	1.81	1.96	2.05	2.50	2.06	2.95	
$\rho$	$\omega$	$\phi$	$J/\psi$	$K^*$	$D^*$	$D_s^*$	$B^*$	$B_s^*$	$\Upsilon$
0.61	0.50	0.91	1.93	0.75	1.51	1.71	1.76	1.71	2.96

Our form factors are represented as three-fold integrals which are calculated by using NAG routines. The results of our numerical calculations are well approximated by the parametrization

$$F(q^2) = \frac{F(0)}{1 - as + bs^2}, \quad s = \frac{q^2}{m_1^2}. \tag{25}$$

We consider the following weak transitions:  $b - u$  (charged current), and  $b - d$  and  $b - s$  (flavor-changing neutral currents). The values of  $F(0)$ ,  $a$ , and  $b$  are listed in Table II.



$q_1 - q_2$	$B - S$	$\Lambda_S = 0.8$			$\Lambda_S = 1.5$		
		$F_+(0)$	$a_+$	$b_+$	$F_+(0)$	$a_+$	$b_+$
$b - u$	$B_d^0 - a_0^+(980)$	0.144	1.624	0.585	0.192	1.433	0.381
$b - u$	$B_s^0 - K_0^{*+}(800)$	0.138	1.667	0.674	0.274	1.258	0.292
$b - s$	$B_s^0 - f_0(980)$	0.141	1.663	0.651	0.254	1.269	0.262
$b - s$	$B_d^0 - K_0^{*0}(800)$	0.191	1.348	0.407	0.306	0.988	0.108
$b - d$	$B_d^0 - f_0(500)$	0.120	1.448	0.485	0.210	1.067	0.155

$q_1 - q_2$	$B - S$	$\Lambda_S = 0.8$			$\Lambda_S = 1.5$		
		$-F_-(0)$	$a_-$	$b_-$	$-F_-(0)$	$a_-$	$b_-$
$b - u$	$B_d^0 - a_0^+(980)$	0.049	2.144	1.196	0.089	1.723	0.688
$b - u$	$B_s^0 - K_0^{*+}(800)$	0.138	1.727	0.734	0.268	1.291	0.310
$b - s$	$B_s^0 - f_0(980)$	0.140	1.761	0.755	0.253	1.320	0.295
$b - s$	$B_d^0 - K_0^{*0}(800)$	0.199	1.406	0.457	0.296	1.032	0.129
$b - d$	$B_d^0 - f_0(500)$	0.116	1.504	0.536	0.191	1.110	0.180

$q_1 - q_2$	$B - S$	$\Lambda_S = 0.8$			$\Lambda_S = 1.5$		
		$F_T(0)$	$a_T$	$b_T$	$F_T(0)$	$a_T$	$b_T$
$b - s$	$B_s^0 - f_0(980)$	0.165	1.680	0.667	0.285	1.276	0.257
$b - s$	$B_d^0 - K_0^{*0}(800)$	0.206	1.367	0.423	0.306	1.005	0.113
$b - d$	$B_d^0 - f_0(500)$	0.124	1.460	0.496	0.203	1.080	0.159

TABLE II: The parameters of the fitted transition form factors  $F(q^2)$  at  $\Lambda_S = 0.8$  GeV and  $\Lambda_S = 1.5$  GeV.

## IV Numerical results and discussion

We use the following set of SM parameters:  $G_F = 1.16637 \times 10^{-5}$  GeV $^{-2}$ ,  $\hat{m}_c = 1.27$  GeV,  $\hat{m}_b = 4.19$  GeV,  $\hat{m}_t = 173.8$  GeV,  $m_W = 80.41$  GeV,  $\sin^2 \theta_W = 0.2233$ ,  $\lambda_t = |V_{ts} V_{tb}^\dagger| = 0.041$ ,  $|V_{ub}| = 0.00413$ , a scale parameter  $\mu = \hat{m}_b$ , and the Wilson coefficients  $C_1 = -0.248$ ,  $C_2 = 1.107$ ,  $C_3 = 0.011$ ,  $C_4 = -0.026$ ,  $C_5 = 0.007$ ,  $C_6 = -0.031$ ,  $C_7^{\text{eff}} = -0.313$ ,  $C_9 = 4.344$ , and  $C_{10} = -4.669$ . We take the average values of the hadron and lepton masses and the  $B_d(B_s)$ -meson lifetimes from Ref. [24].

All model parameters are fixed by fitting the experimental data in our previous papers (see Refs. [49–51, 54]). Their numerical values are shown in Eq. (24) and Table I. The only new parameter is  $\Lambda_S$  which characterizes the size of the scalar mesons. We allow this parameter to vary in a relatively large interval,  $\Lambda_S \in [0.8, 1.5]$  GeV.

In Figs. 3 and 4, we plot our calculated  $F_+(q^2)$  and  $F_T(q^2)$  form factors in the entire kinematical range  $0 \leq q^2 \leq q_{\text{max}}^2$ . Since the behavior of  $-F_-(q^2)$  is very similar to that of  $F_+(q^2)$ , we do not display them. One can see that the form factors are more sensitive to the choice of  $\Lambda_S$  at small  $q^2$  and less so near zero recoil.

We are going to explore the influence of the intermediate scalar  $K_0^*$  meson on the angular decay distribution of the cascade decay  $B \rightarrow K\pi + \mu^+\mu^-$ . Therefore, we give the maximum values of the form factors in Table III and the branching ratios in Table IV obtained for  $\Lambda_S = 1.5$  GeV. The results for the  $e$  mode are almost identical to those of the  $\mu$  mode and will not be shown separately. Since the ratio  $|V_{td}|/|V_{ts}| \approx 0.21$  is relatively small we do not show the branching ratios of the decays with the  $b - d$  transition. We compare the obtained results with those from other approaches. One can see that our values for the branching ratios are almost half of those from other approaches.

Let us briefly discuss the impact of the scalar resonance  $K_0^*$  on  $B \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$  decay. As is well known, the narrow  $K^*(892)$  vector resonance is described by a Breit-Wigner parametrization and the given cascade  $B$  decay can be calculated by using the narrow-width approximation. But this is not true in the case of the broad scalar  $K_0^*(800)$  meson. There are several parametrizations of the  $K - \pi$  line shapes in the literature; see, for instance, the discussion in Ref. [11]. For the time being we will use the parametrization accepted in Ref. [12], the integrated value of which

$B - S$	$F(0)$	This work	[36]	[34]	[18]	[31]	[32]	[55]
$B_d^0 - a_0^+(980)$	$F_+(0)$	0.192	0.58	0.56				
$B_s^0 - K_0^{*+}(800)$	$F_+(0)$	0.274	0.44	0.53				
$B_s^0 - f_0(980)$	$F_+(0)$	0.254	0.45	0.44	0.19	0.35	0.12	0.40
	$F_T(0)$	0.285	0.60	0.58	0.23	0.40	-0.08	
$B_d^0 - K_0^{*0}(800)$	$F_+(0)$	0.306	0.50	0.46				
	$F_T(0)$	0.306	0.67	0.58				
$B_d^0 - f_0(500)$	$F_+(0)$	0.210						
	$F_T(0)$	0.203						

TABLE III: The values of the form factors at  $q^2 = 0$  in the covariant quark model ( $\Lambda_S = 1.5$  GeV) and other approaches.

Decay modes	Branching fractions			
	This work ( $\Lambda_S = 1.5$ GeV)	[37]	[18]	[31]
$B_d^0 \rightarrow a_0^+(980)\mu^-\bar{\nu}_\mu$	$0.52 \times 10^{-4}$	$(2.74 \pm 0.40) \times 10^{-4}$		$1.84 \times 10^{-4}$
$B_d^0 \rightarrow a_0^+(980)\tau^-\bar{\nu}_\tau$	$0.11 \times 10^{-4}$	$(1.31 \pm 0.23) \times 10^{-4}$		$1.01 \times 10^{-4}$
$B_s^0 \rightarrow K_0^{*+}(800)\mu^-\bar{\nu}_\mu$	$1.23 \times 10^{-4}$	$(2.06 \pm 0.31) \times 10^{-4}$		$1.42 \times 10^{-4}$
$B_s^0 \rightarrow K_0^{*+}(800)\tau^-\bar{\nu}_\tau$	$0.25 \times 10^{-4}$	$(1.07 \pm 0.19) \times 10^{-4}$		$0.88 \times 10^{-4}$
$B_d^0 \rightarrow K_0^{*0}(800)\mu^+\mu^-$	$3.47 \times 10^{-7}$	$(7.31 \pm 1.21) \times 10^{-7}$		
$B_d^0 \rightarrow K_0^{*0}(800)\tau^+\tau^-$	$0.61 \times 10^{-7}$	$(1.33 \pm 0.36) \times 10^{-7}$		
$B_s^0 \rightarrow f_0(980)\mu^+\mu^-$	$2.45 \times 10^{-7}$	$(5.14 \pm 0.78) \times 10^{-7}$	$0.95 \times 10^{-7}$	$5.21 \times 10^{-7}$
$B_s^0 \rightarrow f_0(980)\tau^+\tau^-$	$0.42 \times 10^{-7}$	$(0.74 \pm 0.17) \times 10^{-7}$	$1.1 \times 10^{-7}$	$0.38 \times 10^{-7}$
$B_d^0 \rightarrow K_0^{*0}(800)\bar{\nu}\nu$	$2.53 \times 10^{-6}$	$(6.30 \pm 0.97) \times 10^{-6}$		
$B_s^0 \rightarrow f_0(980)\bar{\nu}\nu$	$1.79 \times 10^{-6}$	$(4.39 \pm 0.63) \times 10^{-6}$	$0.87 \times 10^{-6}$	

TABLE IV: The branching fractions for the semileptonic and rare  $B$  decays into light scalar mesons and lepton pairs.

in the  $K^*$ -resonance region is equal to

$$\int_{(m_{K^*}-\delta_m)^2}^{(m_{K^*}+\delta_m)^2} dm_{K\pi}^2 |L_S(m_{K\pi}^2)|^2 = 0.17, \quad \text{where } \delta_m = 100 \text{ MeV}. \quad (26)$$

Then, we scale the calculated value for the differential decay rate  $d\Gamma(B \rightarrow K_0^*(800)\mu^+\mu^-)$  by this factor and compare it with that for  $B \rightarrow K(892)\mu^+\mu^-$  decay. We display the behavior of the ratio

$$R(q^2) = \frac{2/3 d\Gamma(B \rightarrow K^*(892)\mu^+\mu^-)}{2/3 d\Gamma(B \rightarrow K^*(892)\mu^+\mu^-) + 0.17 d\Gamma(B \rightarrow K_0^*(800)\mu^+\mu^-)} \quad (27)$$

in Fig.5, which may be compared with the finding of Ref. [13]. The integrated ratio (the numerator and denominator are integrated separately in the full kinematical region of  $q^2$ ) gives a size for the  $S$ -wave pollution to the branching ratio of the  $B \rightarrow K^*\ell^+\ell^-$  decay of about 6%.

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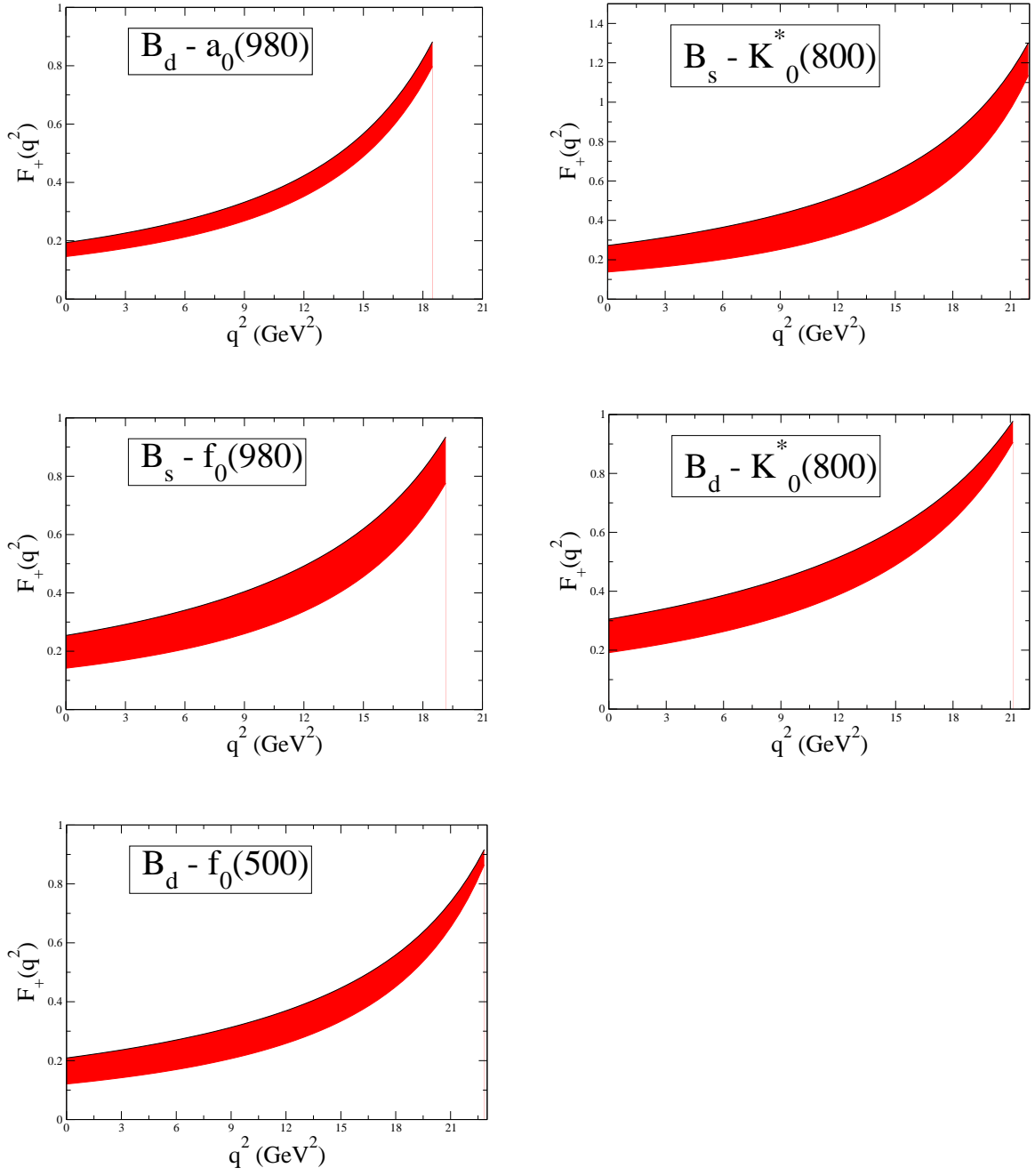


FIG. 3: The  $F_+(q^2)$  form factors for the  $b-u$ ,  $b-d$ , and  $b-s$  transitions. The upper and lower edges correspond to the values  $\Lambda_S = 1.5$  GeV and  $\Lambda_S = 0.8$  GeV, respectively.

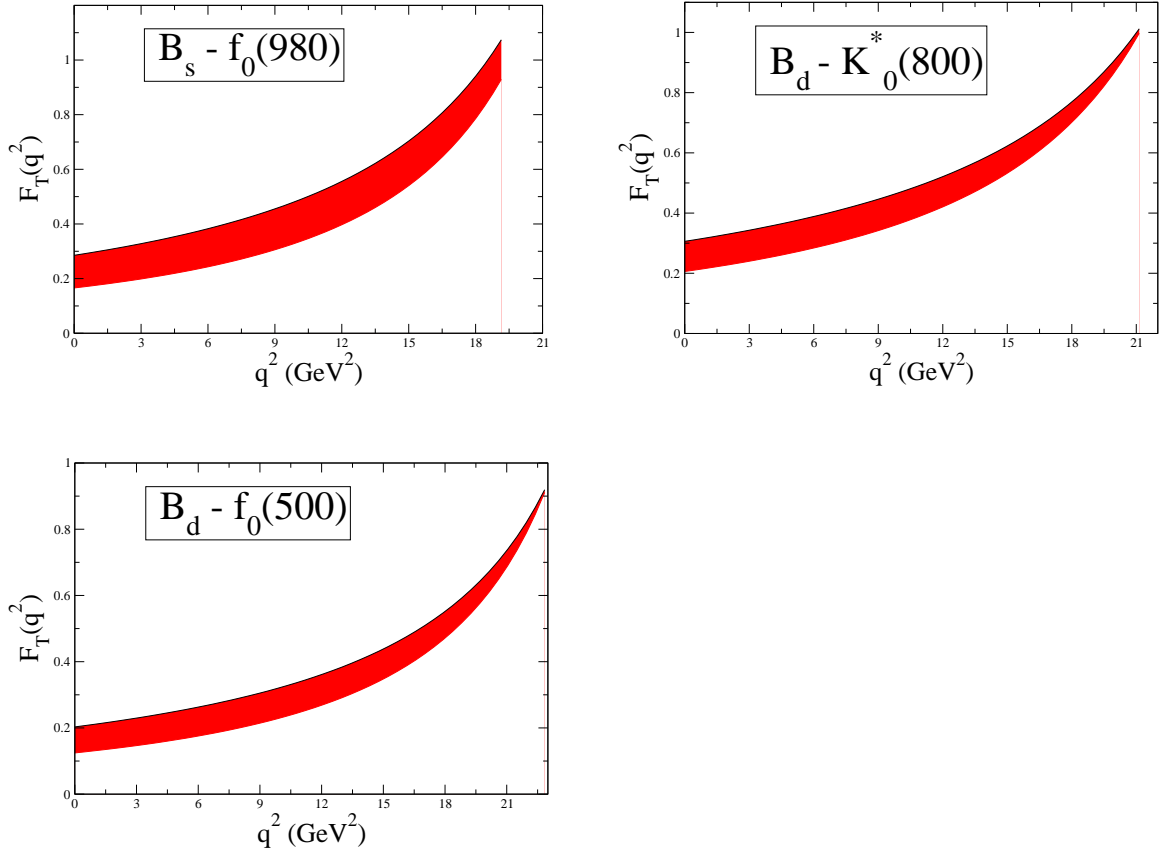


FIG. 4: The  $F_T(q^2)$  form factors for the  $b-u$ ,  $b-d$ , and  $b-s$  transitions. The upper and lower edges correspond to the values  $\Lambda_S = 1.5$  GeV and  $\Lambda_S = 0.8$  GeV, respectively.

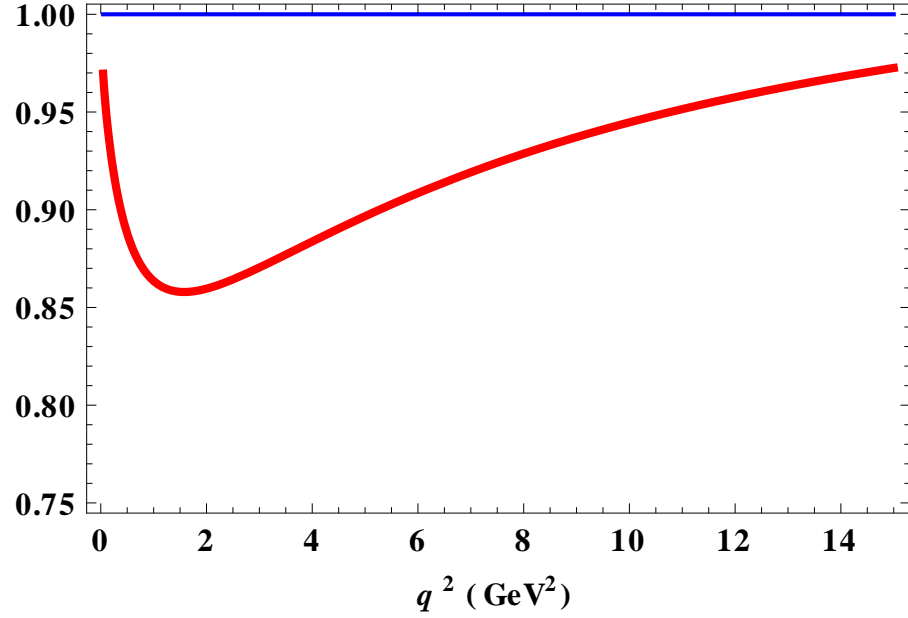


FIG. 5: The ratio of the differential decay rate  $d\Gamma(B \rightarrow K^*(892)(\rightarrow K^0\pi^+)\mu^+\mu^-)$  to the full differential decay rate  $d\Gamma(B \rightarrow K^*(892)(\rightarrow K^+\pi^-)\mu^+\mu^-) + d\Gamma(B \rightarrow K_0^*(\rightarrow K^+\pi^-)\mu^+\mu^-)$ .